

Two trains, A and B, traveling towards each other on parallel tracks, start simultaneously from opposite ends of a 250 mile route. A takes a total of 3 hours to reach the opposite end while B takes a total of 2 hours to reach the opposite end. When train A meets train B during the journey, how far is train A from its starting point?

This question is not difficult but when I give it to my students, I see long, winding solutions, solutions that I find difficult to follow and after looking at a couple, my head starts to spin. Though, admittedly, most of them have the correct answer at the end. But you see, the correct answer alone is not enough. I like to see correct answers along with intuitive, intellectually stimulating methods. The most satisfying is seeing a blank rough sheet and the correct answer together. Rather than relying on the scratch pads and the markers, try and rely on your beautiful minds!

Now, if you haven't tried answering the above given question on your own yet, do it right away before reading the rest of the post.

So what kind of a solution do I like? The following:

Since A takes 3 hours to travel the same distance for which B takes 2 hours, time taken by A and B is in the ratio 3:2 so their speeds must be in the ratio 2:3. Hence, A will cover $\frac{2}{5}$ th of the total distance of 250 miles and B will cover the rest of the $(\frac{3}{5})$ th of the total distance. Therefore, A will be 100 miles from its starting point when A and B meet.

How much time do you think it takes one to think this through? 20-30 seconds at most! It just sounds so simple, doesn't it? But actually it involves a thorough understanding of quite a few fundamental concepts. If you take the time to get comfortable with these concepts, you will be able to save a lot of time on many Arithmetic questions. So let's delve into some basics.

Ratios – One of the most basic and important concepts in the 'just figure it out' methodology.

A ratio gives a measure of the relation between two quantities. It doesn't give either of the two quantities. So score of A : score of B = 4:5 means that for every 4 points A scored, B scored 5 points. (It doesn't mean that A actually scored 4 and B actually scored 5.) To get the actual points scored by the two of them, we would need one of the following (or one of many other) scenarios:

Scenario 1:

Given: A scored 40 points. How many points did B score?

Obviously 50, right? If A scored 10 times 4, B must have scored 10 times 5, right?

Scenario 2:

Given: A and B together scored 90 points. How many points did A score and how many did B score?

If I think in ratio terms, A and B scored 4 and 5 respectively i.e. 9 in all. But actually, they scored 90 in all i.e. 10 times 9. So A must have scored 10 times 4 = 40 and B must have scored 10 times 5 = 50.

Scenario 3:

Given: B scored 10 points more than A. How many points did A score?

B:A is 5:4. For 4 points of A, B scored 1 extra. If B actually scored 10 points more than A, A must have scored 10 times 4 i.e. 40 points (and B must have scored 50 points)

In each of the cases above, you can think of 10 as the common multiplier. It helps you arrive at all the actual values. When you multiply the numbers in ratio terms with the multiplier, you get the actual values. Let us look at another example.

Weight of Adam : Weight of Sally = 7:5. Adam's weight is 133 pounds. What is Sally's weight?

The common multiplier is 19 (because $133/7 = 19$). Hence, Sally's weight is $5 \times 19 = 95$ pounds. The sum of their weights is $(7+5) \times 19 = 228$. The difference of their weights is $(7-5) \times 19 = 38$ pounds

Let us take an actual ratios question from our Arithmetic book now.

The ratio of Blue pens to Red pens is 5:7. When 3 Blue pens are added to the group and 9 Red pens are removed, the ratio of Blue pens to Red pens becomes 3:2. How many Red pens are there after the change?

- (A) 3
- (B) 5
- (C) 9
- (D) 12
- (E) 18

Let us say the multiplier of 5:7 is x .

So there must have been $5x$ Blue pens and $7x$ Red pens before the change.

After the change, number of Blue pens = $5x + 3$, number of Red pens = $7x - 9$

The ratio of these numbers is 3:2 which can also be represented as $3/2$.

$$(5x + 3)/(7x - 9) = 3/2$$

Solving this, we get $x = 3$

After the change, there are $7 \times 3 - 9 = 12$ Red pens

This is the basic framework of ratios. Its uses lie in solving not only 'Ratio Questions' but also 'Weighted Average Questions', 'Distance Speed Time Questions', 'Work Rate Questions' etc. The benefit of using ratios is that you can get rid of the necessity of making and solving equations in many cases. For examples and explanations, check out subsequent posts (that is where I will explain the solution of the trains question given above too).